



NATIONAL BANK OF KAZAKHSTAN

# PARAMETER ESTIMATION FOR DSGE MODELS: PART II

**Monetary Policy Department**

**Economic Study No. 2026-8  
Working Paper**

Adilkhanova Z.

The economic research and analytical reports of the National Bank of the Republic of Kazakhstan (hereinafter – NBRK) are intended to disseminate the results of research conducted by the NBRK, as well as other research studies by its staff. These economic studies are published to foster discussions.

The opinions and judgements presented in this article reflect the views of the authors and should not be construed as representing the views of the NBRK or its management

Parameter estimation for DSGE models: Part II

July 2026

**NBRK – WP – 2026 – 8**

© National Bank of the Republic of Kazakhstan 2026. All rights reserved. Brief excerpts of no longer than one paragraph may be reproduced without the author's permission provided the source is cited

ISSN: 2789-150X

## **Parameter estimation for DSGE models:**

### **Part II**

Zarina Adilkhanova<sup>1</sup>

#### **Abstract**

The paper presents empirical estimates of two parameters required for the analysis of intertemporal household behavior and trade dynamics: the coefficient of relative risk aversion and the elasticity of substitution between domestic and imported goods.

The coefficient of relative risk aversion was estimated using household survey data from the BNS ASPR, covering consumption and subjective well-being indicators for the period 2017-2024. The estimation employed both Maximum Likelihood and Generalized Method of Moments (GMM) techniques. The estimated values range from 1.42 to 1.66, indicating a moderate degree of risk aversion and broadly consistent with findings reported in the international literature.

The elasticity of substitution between imported and domestic goods was estimated using panel data for 56 product categories over the period from January 2020 to September 2025, applying a fixed-effects model. The estimated price elasticity of 0.18 suggests a low degree of substitutability between domestic and imported goods.

**Keywords:** Armington elasticity, Constant Relative Risk Aversion, elasticity of substitution

**JEL Classification:** D12, D81, F14

---

<sup>1</sup> Zarina Adilkhanova – Chief Specialist-Analyst of the Macroeconomic Research and Forecasting Division, Monetary Policy Department, National Bank of the Republic of Kazakhstan.  
Email: [Zarina.Adilkhanova@nationalbank.kz](mailto:Zarina.Adilkhanova@nationalbank.kz)

## Content

|             |                                                                          |           |
|-------------|--------------------------------------------------------------------------|-----------|
| <b>1.</b>   | <b>COEFFICIENT OF RELATIVE RISK AVERSION .....</b>                       | <b>5</b>  |
| <b>1.1.</b> | <b>Introduction .....</b>                                                | <b>5</b>  |
| <b>1.2.</b> | <b>Methodology .....</b>                                                 | <b>5</b>  |
| <b>1.3.</b> | <b>Data.....</b>                                                         | <b>7</b>  |
| <b>1.4.</b> | <b>Results .....</b>                                                     | <b>8</b>  |
| <b>1.5.</b> | <b>Conclusion.....</b>                                                   | <b>10</b> |
| <b>2.</b>   | <b>ELASTICITY OF SUBSTITUTION BETWEEN DOMESTIC AND IMPORTED GOODS ..</b> | <b>11</b> |
| <b>2.1.</b> | <b>Inroduction .....</b>                                                 | <b>11</b> |
| <b>2.2.</b> | <b>Literature review.....</b>                                            | <b>11</b> |
| <b>2.3.</b> | <b>Methodology .....</b>                                                 | <b>12</b> |
| <b>2.4.</b> | <b>Data.....</b>                                                         | <b>13</b> |
| <b>2.5.</b> | <b>Results .....</b>                                                     | <b>14</b> |
| <b>2.6.</b> | <b>Conclusion.....</b>                                                   | <b>16</b> |
| <b>3.</b>   | <b>Literature .....</b>                                                  | <b>17</b> |
| <b>4.</b>   | <b>Appendix .....</b>                                                    | <b>19</b> |

# 1. COEFFICIENT OF RELATIVE RISK AVERSION

## 1.1. Introduction

The Constant Relative Risk Aversion (CRRA) coefficient captures households' sensitivity to uncertainty and is one of the key parameters in intertemporal models of consumer behavior. It determines the extent to which households adjust their consumption in response to higher risk or income volatility and underlies their reaction to changes in intertemporal relative prices, including real interest rates.

In DSGE models, the CRRA coefficient directly affects the elasticity of intertemporal substitution, consumption-smoothing dynamics, the responsiveness of savings to expectations, and the strength of monetary policy transmission channels. Therefore, a reliable estimate of the CRRA parameter is essential for the calibration and quantitative assessment of DSGE models.

Given the significant role of household consumption<sup>2</sup> in Kazakhstan's economy, estimating the CRRA parameter provides valuable insights into household behavior under uncertainty and contributes to a more accurate evaluation of the effectiveness of economic policy measures.

## 1.2. Methodology

The analysis is based on the assumption that the utility of a representative consumer is characterized by utility function  $u_i(c)$  with a constant relative risk aversion. The utility function takes the standard form:

$$u_i(c) = \begin{cases} \frac{c^{1-\sigma} - 1}{1-\sigma}, & \text{if } \sigma \neq 1, \\ \log(c), & \text{if } \sigma = 1, \end{cases}$$

where  $c$  denotes consumption and  $\sigma$  is the coefficient of constant relative risk aversion..

Following Layard et al. (2008) and Gandelman and Hernández-Murillo (2013), it is assumed that individuals are able to consistently evaluate their subjective well-being and that the reported happiness index  $h_{it}$  represents a monotonic transformation of utility:

$$h_{it} = f(\gamma u(c_{it}) + X_{it}\beta + \epsilon_{it}),$$

Under this framework, the coefficient of relative risk aversion,  $\sigma$ , can be identified from observed consumption patterns and subjective well-being measures. The full empirical specification is given by:

$$h_{it} = f\left(\gamma \frac{c_{it}^{1-\sigma} - 1}{1-\sigma}\right) + \beta_1 \text{female} + \beta_2 \text{age} + \beta_3 \text{age}^2 + \beta_4 \text{urban} \\ + \beta_5 \text{income level} + \theta_k \text{location} + \epsilon_{it},$$

where

---

<sup>2</sup> Household consumption expenditure accounts for approximately 87% of total monetary household income on average over the period 2016-2025, according to data from the BNS ASPR.

- $h_{it}$  - denotes the subjective well-being index, which serves as the dependent variable in the model. In this study, three alternative indicators are used as proxies for happiness: life satisfaction, satisfaction with living conditions, and the overall level of happiness. The estimations are conducted separately for each of these indicators;
- *female* - a dummy variable controlling for gender, taking the value of 1 for female respondents and 0 for male respondents;
- *age* denotes the respondent's age;
- *urban* is a dummy variable equal to 1 if the respondent lives in an urban area and 0 if the respondent lives in a rural area;
- *income level* is a dummy variable equal to 1 if the respondent is satisfied with their financial situation and 0 if the respondent is not satisfied or only partially satisfied;
- *location* is a vector of dummy variables controlling for the respondent's place of residence.

The study applies iterative maximization of the likelihood function (MLE) within a nonlinear model to estimate the coefficient of relative risk aversion ( $\sigma$ ).

The estimation algorithm consists of the following steps:

1. First, the log-likelihood function is evaluated over the range of  $\sigma$  from 0 to 3 with a step size of 0.1.
2. Second, the estimates are refined around the identified maximum using a smaller step size of 0.01.
3. At each iteration, an ordered logit model is applied to estimate the model coefficients.
4. To mitigate the influence of outliers, the top and bottom 5% of observations by consumption level are excluded from the sample, following Layard et al. (2008).

In addition, the Generalized Method of Moments (GMM) is used to assess the robustness of the estimates. Since the CRRA parameter enters the model nonlinearly, the study applies a modified approach based on Gandelman and Hernández-Murillo (2013), relying on a monotonic transformation of the utility function:

$$g(c, \sigma) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \quad m(c, \sigma) = \frac{\partial g}{\partial \sigma}$$

For the implementation of GMM, the analytical derivative  $m(c, \sigma)$  with respect to  $\sigma$  is required. Both functions,  $g(c, \sigma)$  and  $m(c, \sigma)$ , are used as moment conditions and instruments, which allows the parameter  $\sigma$  to be properly identified.

The GMM estimation of  $\sigma$  is carried out through several iterative steps to ensure convergence and robustness of the results:

1. At the first stage, an OLS regression of the dependent variable on the transformed utility function  $g(c, \sigma_0)$ , is estimated, assuming an initial value of the risk aversion parameter of approximately  $\sigma_0 \approx 0.5$ . This step is used to construct the initial instruments  $g(c, \sigma_0)$  and  $m(c, \sigma_0)$ .
2. Using the constructed instruments, the first GMM estimation is then performed:

$$3. h_{it} = \gamma g(c_{it}, \sigma) + X_{it}\beta + \epsilon_{it}$$

This step yields an updated estimate,  $\hat{\sigma}_1$ .

4. After obtaining  $\hat{\sigma}_1$ , the instruments are recalculated. This updating procedure helps eliminate the bias that may arise from using inappropriate instruments at the initial stage.
5. The GMM estimation is then repeated until convergence is achieved.

The use of multi-step GMM in addition to MLE estimates provides an independent robustness check for the CRRA parameter, verifies the validity of identification, and allows for more reliable standard errors.

Thus, the combination of the two estimation methods provides a theoretically grounded and statistically robust estimate of the coefficient of constant relative risk aversion for Kazakhstan.

### 1.3. Data

The empirical analysis is based on survey data from the Bureau of National Statistics of the Agency for Strategic Planning and Reforms of the Republic of Kazakhstan (BNS ASPR), which contain information on household consumption, socio-demographic characteristics, and subjective well-being. The data are drawn from the following survey instruments: Forms D-003 and D-004, which provide quarterly cross-sectional surveys of household consumption, and Form D-002, which contains the annual household welfare survey.

The combined sample comprises approximately 93,000 households over the period 2017–2024. Since consumption in the model is defined on an annual basis, the quarterly data from Forms D-003 and D-004 were aggregated to annual values. Aggregation was performed by summing household consumption across all quarters within each calendar year. This procedure ensures consistency in the frequency of all variables and allows the proper use of subjective well-being indicators, which are available only at the annual frequency. Household consumption expenditure was deflated using the Consumer Price Index (CPI) and converted into real terms at 2017 constant prices.

The sample is characterized by the following demographic features:

- **53%** of households reside in urban areas, while **47%** reside in rural areas;
- **59%** of respondents are female and **41%** are male;
- **34%** of respondents report being satisfied with their financial situation, whereas **66%** are either dissatisfied or only partially satisfied;
- **67%** classify their standard of living as “average” (middle class).

Three indicators are used as dependent variables: life satisfaction, satisfaction with household living conditions (available for 2017-2024), and the happiness index (available for 2018-2022) (Table 1.1). These indicators are measured on a scale from 1 to 10, where 1 corresponds to “completely dissatisfied” or “extremely unhappy” and 10

corresponds to “completely satisfied” or “very happy.” In the BNS ASPR questionnaires, the code 89 is used to indicate either “difficult to answer” or “not applicable.” Such observations are excluded from the analysis because they do not contain information on the respondent’s actual level of satisfaction or happiness and may introduce bias into the estimation results.

**Table 1.1. Descriptive Statistics (Full Sample)**

| Variable                             | Mean   | Std. Dev. | Min   | Max      | Number of Observations |
|--------------------------------------|--------|-----------|-------|----------|------------------------|
| Age                                  | 45.54  | 16.53     | 15    | 99       | 93 044                 |
| Annual Consumption, Real KZT         | 953472 | 577581    | 88915 | 21200000 | 93 044                 |
| Happiness Index                      | 8.11   | 1.57      | 1     | 10       | 57 398                 |
| Life Satisfaction Index              | 7.66   | 1.56      | 1     | 10       | 92 882                 |
| Living Conditions Satisfaction Index | 7.56   | 1.61      | 1     | 10       | 92 859                 |

Source: BNS ASPR

#### 1.4. Results

The maximum likelihood estimation results for the coefficient of constant relative risk aversion are highly robust across all three measures of subjective well-being: life satisfaction, satisfaction with living conditions, and self-reported happiness. The estimated values of the coefficient range from 1.42 to 1.66, indicating a moderate degree of risk aversion among households and suggesting that individuals place substantial weight on the trade-off between current and future consumption (Table 1.2).

The introduction of an age restriction (25-65 years) has virtually no effect on the estimates. For example, the estimated value of  $\sigma$  for life satisfaction changes only marginally, from 1.66 to 1.65, confirming the stability of the parameter across alternative sample specifications. The statistical significance of the estimates ranges from the 1% to the 10% level, with most estimates significant at the 5% and 1% levels, indicating both the reliability of the model and the precision of the estimated coefficients.

A comparison across subjective well-being measures reveals that the estimated values of  $\sigma$  for satisfaction with living conditions are somewhat lower (1.42-1.55) than those obtained for life satisfaction and self-reported happiness. This finding may reflect a lower sensitivity of respondents to risk when evaluating their material living conditions relative to broader assessments of life satisfaction or overall happiness. Overall, the results confirm that the population of Kazakhstan exhibits a moderate degree of risk aversion, making these estimates suitable for calibrating the CRRA parameter in intertemporal consumption models and for analyzing household behavior under conditions of economic uncertainty.



**Table 1.2. Estimates of the Constant Relative Risk Aversion Parameter ( $\sigma$ ), MLE**

| <i>Dependent Variable</i>                  | <i>Age Restriction (25-65)</i> | <i>Coefficient <math>\sigma</math></i> | $\chi^2$ | <i>N</i> |
|--------------------------------------------|--------------------------------|----------------------------------------|----------|----------|
| <i>How happy do you consider yourself?</i> | No                             | <b>1.48***</b>                         | 9.51     | 50 404   |
|                                            | Yes                            | <b>1.52***</b>                         | 9.18     | 39 391   |
| <i>Life Satisfaction</i>                   | No                             | <b>1.66*</b>                           | 6.99     | 81 552   |
|                                            | Yes                            | <b>1.65**</b>                          | 5.35     | 63 504   |
| <i>Satisfaction with Living Conditions</i> | No                             | <b>1.42**</b>                          | 4.25     | 81 552   |
|                                            | Yes                            | <b>1.55**</b>                          | 5.15     | 63 504   |

Source: Author's estimates and calculations.

Notes: Statistical significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . Estimation method: Maximum Likelihood Estimation (MLE), Ordered Logit. Dependent variables are measured on a scale from 1 to 10.

Table 1.3 presents the estimates of the constant relative risk aversion parameter obtained using the Generalized Method of Moments (GMM). Across all specifications, the coefficient is statistically significant at the 1% level and lies within a narrow range of 1.43-1.49, indicating a high degree of robustness with respect to both the choice of dependent variable and the application of age restrictions. The estimated values are comparable to those obtained using the MLE approach and fall within the broader range of 1.42-1.66, highlighting the consistency of the results across the two estimation methods and reinforcing the reliability of the estimated risk aversion parameter. From an economic perspective, these findings suggest that individuals generally prefer stable and predictable environments but do not avoid uncertainty altogether. Households appear willing to accept risk provided that the expected benefits are sufficiently high to compensate for potential losses.

**Table 1.3. Estimates of the Constant Relative Risk Aversion Parameter ( $\sigma$ ), GMM**

| <i>Dependent Variable</i>                  | <i>Age Restriction (25-65)</i> | <i>Coefficient <math>\sigma</math></i> | <i>SE</i> | <i>N</i> |
|--------------------------------------------|--------------------------------|----------------------------------------|-----------|----------|
| <i>How happy do you consider yourself?</i> | No                             | <b>1.46***</b>                         | 0.004     | 50 463   |
|                                            | Yes                            | <b>1.45***</b>                         | 0.005     | 39 437   |
| <i>Life Satisfaction</i>                   | No                             | <b>1.46***</b>                         | 0.009     | 81 294   |
|                                            | Yes                            | <b>1.49***</b>                         | 0.012     | 63 572   |
| <i>Satisfaction with Living Conditions</i> | No                             | <b>1.43***</b>                         | 0.029     | 81 294   |
|                                            | Yes                            | <b>1.44***</b>                         | 0.006     | 63 587   |

Source: Author's estimates and calculations.

Notes: Statistical significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

Estimation method: GMM

The literature reports considerable variation in estimates of the coefficient of relative risk aversion across countries and estimation methodologies. In the seminal

study by Szpiro and Outreville (1988), the coefficient ranges from 1 to 5, with an average value of 2.89 across a sample of 31 countries, reflecting a relatively high degree of risk aversion in international data for that period. Gandelman and Hernández-Murillo (2013) find that estimates based on cross-sectional data are generally close to 1, corresponding to a logarithmic utility function, whereas estimates derived from panel data tend to be somewhat higher. In a subsequent study, Gandelman and Hernández-Murillo (2015) report values ranging from 0 to 3, with an average of 0.98 across a sample of 75 advanced and emerging economies, including the Kyrgyz Republic (1.81), Azerbaijan (1.85), Armenia (0.57), Russia (0.65), Tajikistan (1.19), and Uzbekistan (2.96). These findings highlight substantial cross-country heterogeneity in the degree of risk aversion.

The CRRA estimates obtained in this study fall within a similar range and are broadly consistent with the international literature. Their proximity to the values reported in previous empirical studies supports the internal consistency of our results and suggests that the estimated models adequately capture the behavioral characteristics of households. Such comparability enhances the credibility of the findings and supports the applicability of the adopted approach for analyzing risk preferences.

The CRRA parameter plays a central role in DSGE models as well as in other dynamic macroeconomic frameworks, including studies of fiscal and pension policies, as it directly affects the characterization of intertemporal preferences and household responses to economic shocks.

### **1.5. Conclusion**

The estimated values of the coefficient of constant relative risk aversion for Kazakhstan indicate a moderate degree of risk aversion among households, ranging from 1.42 to 1.66 (MLE) and from 1.43 to 1.49 (GMM). The results are robust to the choice of subjective well-being indicator, the application of age restrictions, and the estimation method employed, confirming the reliability of the findings.

The estimated values fall within the range reported in the international empirical literature, suggesting consistency with existing evidence on household risk preferences. This comparability supports the validity of the estimates and confirms their suitability for calibrating DSGE models and other intertemporal macroeconomic frameworks.

## **2. ELASTICITY OF SUBSTITUTION BETWEEN DOMESTIC AND IMPORTED GOODS**

### **2.1. Introduction**

Relative price movements influence how consumers allocate spending between domestically produced and imported goods. Examining this response provides insight into the degree of substitutability between goods by country of origin and helps characterize the structure of aggregate demand.

To capture this mechanism quantitatively, the elasticity of substitution between domestic and imported goods - commonly referred to as the Armington elasticity - is employed. This parameter measures the responsiveness of the relative demand for domestic and imported goods to changes in their relative prices.

A high elasticity implies that consumers readily substitute between domestic and imported products, increasing competitive pressures and limiting firms' pricing power. Conversely, a low elasticity suggests the presence of product differentiation arising from quality differences, brand preferences, consumer tastes, or institutional and logistical constraints.

Following the framework of Armington (1969), domestic and imported goods are treated as imperfect substitutes, even within the same product category. This assumption is typically represented by a Constant Elasticity of Substitution (CES) aggregator, in which goods are differentiated by country of origin and enter the consumption bundle as distinct varieties.

The objective of this section is to estimate the elasticity of substitution between domestic and imported goods for broad product categories (food and non-food goods) in Kazakhstan. The resulting estimates provide a quantitative measure of the responsiveness of demand to relative price changes and form the basis for subsequent analysis.

Estimating this parameter is particularly important for macroeconomic modeling. The Armington elasticity is a key structural parameter in Dynamic Stochastic General Equilibrium (DSGE) models, determining the extent to which domestic and imported goods can be substituted for one another. As such, it influences the dynamics of consumption, inflation, and output, as well as the transmission of relative price and external shocks throughout the economy. The estimates obtained in this section can therefore be used for the calibration and quantitative analysis of DSGE models.

### **2.2. Literature review**

The concept of goods differentiated by country of origin was first introduced by Armington (1969). He argued that goods belonging to the same product category but produced in different countries are not perfect substitutes and proposed a Constant Elasticity of Substitution (CES) functional form to quantify the degree of substitutability between them.

Subsequently, the Armington framework became a central component of international trade models and general equilibrium models. Important contributions to its theoretical development were made by Anderson and van Wincoop (2003) and Feenstra (2018).

Empirical studies report a wide range of estimates for the elasticity of substitution. For example, Hillberry and Hummels (2013) show that CES elasticity estimates vary substantially depending on the data, estimation methods, and level of product aggregation. Feenstra et al. (2018) proposed improved identification techniques based on panel data.

Classic applications of the Armington framework can be found in Dixon, Jerie and Rimmer (2016), Zhai (2008), Lloyd and Zhang (2006), Zhang and Verikios (2006), Shiells and Reinert (1993), and Bandara (1991). The existing empirical literature reveals considerable variation in Armington elasticity estimates across countries (Kenneth et al., 2021; Bajzik et al., 2019; Olekseyuk and Schürenberg-Frosch, 2016).

Overall, the literature suggests that CES elasticity estimates differ substantially across both countries and product categories. Consequently, obtaining an empirical estimate of the elasticity of substitution for a particular economy constitutes an important research task in its own right.

### 2.3. Methodology

Following the Armington (1969) framework and its modern extensions (Hillberry and Hummels, 2013; Feenstra et al., 2018), consumer preferences are represented by a Constant Elasticity of Substitution (CES) utility function:

$$U(Q_D, Q_M) = \left[ \beta Q_D^{\frac{\gamma-1}{\gamma}} + (1-\beta) Q_M^{\frac{\gamma-1}{\gamma}} \right]^{\gamma/(\gamma-1)}$$

where:

- $Q_D$  denotes consumption of domestically produced goods;
- $Q_M$  denotes consumption of imported goods;
- $\beta$  is the preference share parameter between domestic and imported goods;
- $\gamma$  is the elasticity of substitution between domestic and imported goods.

Consumers minimize total expenditure  $E = P_D Q_D + P_M Q_M$  subject to a given level of utility, where  $P_D$  denotes the price of domestically produced goods and  $P_M$  denotes the price of imported goods. The first-order condition yields the equilibrium relationship between relative consumption and relative prices:

$$\frac{Q_M}{Q_D} = \left[ \frac{1-\beta}{\beta} * \frac{P_D}{P_M} \right]^{\gamma}$$

Taking logarithms gives a specification suitable for econometric estimation:

$$\log \left( \frac{Q_M}{Q_D} \right) = \gamma \log \left( \frac{1-\beta}{\beta} \right) + \gamma \log \left( \frac{P_D}{P_M} \right) + \epsilon,$$

Where  $\epsilon$  is the error term.

From this equation, it follows that the coefficient on  $\log\left(\frac{P_D}{P_M}\right)$  corresponds to the elasticity of substitution parameter.

A high value of  $\gamma$  implies that domestic and imported goods are perceived as close substitutes and can be readily replaced by one another, meaning that changes in relative prices have a strong effect on the composition of demand. Conversely, a low value of  $\gamma$  indicates limited substitutability and a stronger consumer preference for goods of a particular origin.

Following Sauquet et al. (2011), a lagged dependent variable is included in the specification to capture the long-run relationship between quantities and prices and to improve the explanatory power of the model:

$$\log\left(\frac{Q_{it}^M}{Q_{it}^D}\right) = \delta + \alpha \log\left(\frac{Q_{it-1}^M}{Q_{it-1}^D}\right) + \gamma \log\left(\frac{P_{it}^D}{P_{it}^M}\right) + \gamma KEI_t + v_{it} + \mu_t + \epsilon_{it},$$

where  $\delta$  is a constant term,  $KEI$  is the Short-Term Economic Indicator,  $v_{it}$  denotes product fixed effects;  $\mu_t$  denotes time fixed effects,  $\epsilon_{it}$  is the idiosyncratic error term. To account for unobserved time-invariant heterogeneity across product categories, the analysis employs a fixed-effects panel data model. In addition, time fixed effects are included to control for macroeconomic fluctuations affecting all product categories simultaneously. However, in short panels, dynamic fixed-effects estimators may suffer from inconsistency. To address this issue, the analysis also applies the Arellano-Bond and Arellano-Bover Generalized Method of Moments (GMM) estimators. As shown by Arellano and Bond (1991), lagged first differences can be used as instrumental variables to address endogeneity and unobserved individual effects. Arellano and Bover (1995) subsequently extended this framework by using both lagged differences and lagged levels as instruments. Finally, Labra and Torrecillas (2018) demonstrate that the two-step GMM estimator is generally more efficient than the one-step alternative.

## 2.4. Data

The empirical analysis is based on monthly official statistics published by the BNS ASPR. All time series were tested and seasonally adjusted to remove regular intra-year fluctuations and ensure comparability of the data over time.

Table 2.1 presents the variables used to estimate the elasticity parameter. Data on import volumes and domestic production volumes are obtained from the publication “Resources and Uses of Selected Products and Raw Materials in the Republic of Kazakhstan.” Price indicators include the import price index, compiled from the publication “Price Indices of Export Deliveries and Import Receipts of Goods and Products of the Republic of Kazakhstan,” as well as producer price indices for industrial and agricultural products. To control for overall macroeconomic conditions, the model incorporates the Short-Term Economic Indicator (KEI) published by BNS ASPR.

The initial sample consisted of 69 food and non-food product categories, which together account for more than 80% of the food basket and approximately 60% of the non-food basket of the Consumer Price Index (CPI). Further details are provided in

Table A1 of the Appendix. Due to data availability constraints for a number of product categories, the final sample was reduced accordingly. As a result, the analysis is based on a balanced panel covering 56 products.

The final dataset spans the period from January 2020 to September 2025 and is structured as a monthly panel dataset at the product level.

**Table 2.1. Data Used in the Analysis**

| Variables                     | Notation | Data Source                                                                                                          |
|-------------------------------|----------|----------------------------------------------------------------------------------------------------------------------|
| Import Volume                 | $Q_M$    | Resources and Uses of Selected Products and Raw Materials in the Republic of Kazakhstan, BNS ASPR                    |
| Domestic Production Volume    | $Q_D$    | Resources and Uses of Selected Products and Raw Materials in the Republic of Kazakhstan, BNS ASPR                    |
| Import Price Index            | $P_M$    | Price Indices of Export Deliveries and Import Receipts of Goods and Products of the Republic of Kazakhstan, BNS ASPR |
| Producer Price Index          | $P_D$    | Industrial Producer Price Index and Agricultural Producer Price Index, BNS ASPR                                      |
| Short-Term Economic Indicator | $KEI$    | BNS ASPR                                                                                                             |

## 2.5. Results

The estimation results are presented in Table 2.2. The dependent variable is the logarithm of the ratio of import volume to domestic production volume. The model is estimated using product-level fixed effects and time fixed effects, allowing for the control of unobserved heterogeneity across product categories as well as common macroeconomic shocks.

The coefficient on the lagged dependent variable is positive and statistically significant at the 1% level, indicating substantial persistence in the relationship between imports and domestic production. The estimated coefficient of 0.508 suggests that changes in the supply structure are highly persistent and adjust only gradually over time.

**Table 2.2. Fixed-Effects Regression Results**

| Variables                                      | $\log\left(\frac{Q_t^M}{Q_t^D}\right)$ |
|------------------------------------------------|----------------------------------------|
| $\log\left(\frac{Q_{t-1}^M}{Q_{t-1}^D}\right)$ | 0.508***<br>(0.014)                    |
| $\log\left(\frac{P_t^D}{P_t^M}\right)$         | <b>0.182***</b><br>(0.058)             |

|                           |                   |
|---------------------------|-------------------|
| KEI                       | -0.029<br>(0.047) |
| Time Fixed Effects        | Yes               |
| Fixed Effects             | Yes               |
| Constant                  | 2.695<br>(4.622)  |
| <i>N</i>                  | 3795              |
| <i>Adj. R<sup>2</sup></i> | 0.249             |

Source: Author's calculations.

Notes: Statistical significance levels: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

Estimation method: Fixed-effects estimator.

The coefficient on the relative price variable can be interpreted as an estimate of the elasticity of substitution between imported and domestically produced goods within the standard CES framework. The estimated coefficient of 0.182 is positive and statistically significant.

The magnitude of the estimate indicates a low elasticity of substitution between imported and domestic products. This suggests that imports and domestic production within the product groups considered behave as imperfect substitutes, with only limited adjustments in relative quantities in response to changes in relative prices.

This finding is consistent with the assumptions of Armington-type models and reflects the presence of structural constraints, including technological specialization, quality differences, logistical barriers, and institutional factors that limit the rapid reallocation of demand and supply. Another factor contributing to the low elasticity of substitution may be the limited domestic production base. Some goods are either not produced domestically or are produced in insufficient quantities, reducing the scope for import substitution even when relative prices change.

The low elasticity estimate also implies that changes in price competitiveness have only a moderate effect on the relationship between imports and domestic production in the short run. Compared with previous empirical studies, the estimated value lies in the lower part of the distribution of reported elasticity estimates, although it remains within the range documented in the literature.

In particular, Reinert and Roland-Holst (1992), estimating Armington elasticities for 163 manufacturing and extractive industries in the United States, report statistically significant estimates ranging from 0.14 to 3.49. More recent studies employing detailed datasets and alternative estimation approaches generally find higher median elasticities of substitution. For example, Gallaway, McDaniel, and Rivera (2000) report significant long-run estimates ranging from 0.52 to 4.83, while the estimates reported by Broda and Weinstein (2006), Hertel et al. (2007), Caliendo and Parro (2015), and Ossa (2015) are substantially higher, particularly when using systems of equations, trade-cost frameworks, and highly disaggregated product classifications.

The elasticity estimate obtained in this study likely reflects the specific characteristics of Kazakhstan's domestic market structure, as well as the short-run

nature of the empirical specification. In this regard, the findings are consistent with earlier studies emphasizing limited substitutability between imported and domestically produced goods at the level of broad product categories.

The model's coefficient of determination (adjusted  $R^2 = 0.249$ ) indicates moderate explanatory power, which is typical for fixed-effects panel regressions at the product level. The estimation sample consists of 3,795 observations.

To assess the robustness of the results, dynamic panel estimators based on the Arellano-Bond and Arellano-Bover approaches were also employed. However, the resulting coefficients were statistically insignificant. A likely explanation is the relatively short time dimension of the panel, combined with the limited number of available instruments, which reduces the efficiency of GMM estimation.

## **2.6. Conclusion**

The Armington elasticity of substitution plays a central role in the analysis of trade policy and is a key parameter for the calibration of DSGE models. Despite the absence of a single established approach to estimating this parameter in the literature, the results obtained for Kazakhstan indicate a relatively low but statistically significant elasticity of substitution between imported and domestically produced goods.

This finding suggests that imports and domestic production act as imperfect substitutes and that relative price signals have only a limited effect on the reallocation of supply structures in the short run. The estimated elasticity is consistent with the characteristics of Kazakhstan's domestic market and supports the view that substitution between imported and domestic goods is constrained by structural factors.

The estimates obtained in this study can be used for the calibration of macroeconomic models and for the analysis of trade and price shocks.



### 3. Literature

Anderson, J. E., & Van Wincoop, E. (2003). Gravity with gravitas: A solution to the border puzzle. *American Economic Review*, 93(1), 170–192.

Arellano, M., & Bond, S. (1991). Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations. *Review of Economic Studies*, 58(2), 277–297.

Arellano, M., & Bover, O. (1995). Another look at the instrumental variable estimation of error-components models. *Journal of Econometrics*, 68(1), 29–51.

Armington, P. (1969). A theory of demand for products distinguished by place of production. *IMF Staff Papers*, 16(1), 159–176.

Bajzik, J., Havranek, T., Irsova, Z., & Schwarz, J. (2019). Estimating the Armington elasticity: The importance of data choice and publication bias. Charles University Prague, Faculty of Social Sciences, Institute of Economic Studies, Working Paper IES 2019/19 (revised July 2019).

Bandara, J. S. (1991). Computable general equilibrium models for development policy analysis in LDCs. *Journal of Economic Surveys*, 5(1), 3–69. <https://doi.org/10.1111/j.1467-6419.1991.tb00126.x>

Broda, C. and Weinstein, D. (2006). Globalization and the Gains from Variety, *Quarterly Journal of Economics* 121: 541-585.

Caliendo, L. and Parro, F. (2015). Estimates of the Trade and Welfare Effects of NAFTA, *The Review of Economic Studies* 82: 1-44.

Dixon, P. B., Jerie, M., & Rimmer, M. T. (2016). Modern trade theory for CGE modelling: The Armington, Krugman and Melitz models. *Journal of Global Economic Analysis*, 1(1), 1–110. <https://doi.org/10.21642/JGEA.010101AF>

Feenstra, R. C., Luck, P., Obstfeld, M., & Russ, K. N. (2018). In search of the Armington elasticity. *The Review of Economics and Statistics*, 100(1), 135–150.

Gandelman, N., & Hernández-Murillo, R. (2013). What do happiness and health satisfaction data tell us about relative risk aversion? *Journal of Economic Psychology*, 39, 301-312.

Gandelman, N., & Hernández-Murillo, R. (2015). Risk aversion at the country level. *Available at SSRN 2646134*.

Gallaway, M.P., McDaniel, C.A., Rivera, S.A., 2000. Short-run and long-run industry-level estimates of U.S. Armington elasticities, *North American Journal of Economics and Finance*, forthcoming

Hertel, T., Hummels, D., Ivanic, M. and Keeney, R. (2007). How Confident can we be of CGE-based Assessments of Free Trade Agreements?, *Economic Modelling* 24: 611-635.

Hillberry, R., & Hummels, D. (2013). Trade elasticity parameters for a computable general equilibrium model. In P. B. Dixon & D. W. Jorgenson (Eds.), *Handbook of Computable General Equilibrium Modeling* (Vol. 1, pp. 1213–1269). North-Holland, Oxford: Elsevier.

- Labra, R., & Torrecillas, C. (2018). Estimating dynamic panel data: A practical approach to perform long panels. *Revista Colombiana de Estadística*, 41(1), 31–52.
- Layard, R., Mayraz, G., & Nickell, S. (2008). The marginal utility of income. *Journal of Public Economics*, 92(8-9), 1846-1857
- Lloyd, P. J., & Zhang, X. (2006). The Armington model. Productivity Commission Staff Working Paper, Melbourne, January.
- Olekseyuk, Z., & Schurenberg-Frosch, H. (2016). Are Armington elasticities different across countries and sectors? A European study. *Economic Modelling*, 55, 328–342. <https://doi.org/10.1016/j.econmod.2016.02.018>
- Ossa, R. (2015). Why Trade Matters After All, *Journal of International Economics* 97: 266- 277
- Reinert, K.A., Roland-Holst, D.W., 1992. Armington elasticities for United States manufacturing sectors, *Journal of Policy Modeling*, 14(5), 631-39.
- Sauquet, A., Lecocq, F., Delacote, P., Caurila, S., Barkaoui, A., & Garcia, S. (2011). Estimating Armington elasticities for sawn wood and application to the French forest sector model. *Resource and Energy Economics*, 33(4), 771–781.
- Shiells, C. R., & Reinert, K. A. (1993). Armington models and terms-of-trade effects: Some econometric evidence for North America. *Canadian Journal of Economics*, 26(2), 299–316. <https://doi.org/10.2307/135909>
- Szpiro, G. G., & Outreville, J. F. (1988). Relative risk aversion around the world: Further results. *Journal of Banking & Finance*, 6(1, Supplement), 127-128.
- Zhai, F. (2008). Armington meets Melitz: Introducing firm heterogeneity in a global CGE model of trade. *Journal of Economic Integration*, 23(3), 575–604. <https://doi.org/10.11130/jei.2008.23.3.575>
- Zhang, X. (2006). Armington elasticities and terms of trade effects in global CGE models. Productivity Commission Staff Working Paper 0601, Government of Australia.
- Zhang, X., & Verikios, G. (2006). Armington parameter estimation for a computable general equilibrium model: A database consistent approach. *Economics Discussion Paper 06-10*, University of Western Australia.
- Clements, K. W., Mariano, M. J. M., & Verikios, G. (2021). Foreign-domestic substitution, import penetration and CGE modelling. *Applied Economics*, 53(35), 4080–4099. <https://doi.org/10.1080/00036846.2021.1897072>

## 4. Appendix

Table A1. List of Products

| No. | Product                                                                                                                             |
|-----|-------------------------------------------------------------------------------------------------------------------------------------|
| 1   | Meat, poultry meat, and edible offal                                                                                                |
| 2   | Other edible meat and meat offal products, salted, in brine, dried or smoked (excluding pork and beef); edible meat meal and powder |
| 3   | Sausages and similar products of meat, meat offal, or animal blood                                                                  |
| 4   | Processed and preserved fish, crustaceans, and mollusks                                                                             |
| 5   | Fruit and vegetable juices                                                                                                          |
| 6   | Processed and preserved vegetables, excluding potatoes                                                                              |
| 7   | Processed and preserved fruits and nuts                                                                                             |
| 8   | Vegetable oils                                                                                                                      |
| 9   | Margarine and similar products                                                                                                      |
| 10  | Processed liquid milk and cream                                                                                                     |
| 11  | Butter                                                                                                                              |
| 12  | Cheese and curd                                                                                                                     |
| 13  | Condensed milk and cream, with or without added sugar or other sweetening matter, not in solid form                                 |
| 14  | Yogurt and other fermented or cultured milk and cream                                                                               |
| 15  | Ice cream and edible ice (including sherbet and ice lollies), excluding mixes and bases for ice cream production                    |
| 16  | Flour                                                                                                                               |
| 17  | Cereals, including rice                                                                                                             |
| 18  | Bread; cakes and confectionery products; other bakery products containing sweetening matter                                         |
| 19  | Crispbread, biscuits, pastries, and preserved confectionery products                                                                |
| 20  | Pasta, noodles, couscous, and similar farinaceous products                                                                          |
| 21  | Sugar                                                                                                                               |
| 22  | Chocolate and sugar confectionery                                                                                                   |
| 23  | Tea and coffee                                                                                                                      |
| 24  | Vinegar, sauces, mixed seasonings, mustard flour and powder, prepared mustard                                                       |
| 25  | Table salt                                                                                                                          |
| 26  | Vodka and spirits                                                                                                                   |
| 27  | Cognac and cognac-based beverages                                                                                                   |
| 28  | Wine (excluding cider and grape must)                                                                                               |
| 29  | Beer, excluding brewing residues and waste                                                                                          |
| 30  | Mineral water and soft drinks                                                                                                       |
| 31  | Cigarettes and tobacco products                                                                                                     |
| 32  | Eggs                                                                                                                                |
| 33  | Cotton fabrics                                                                                                                      |
| 34  | Outerwear                                                                                                                           |
| 35  | Footwear, excluding sports, protective, and orthopedic footwear                                                                     |
| 36  | Portland cement (excluding white cement)                                                                                            |
| 37  | Gypsum                                                                                                                              |
| 38  | Paints, varnishes, related products, artists' colors, and printing inks                                                             |
| 39  | Ceramic tiles and slabs                                                                                                             |

|    |                                                                                                                                           |
|----|-------------------------------------------------------------------------------------------------------------------------------------------|
| 40 | Linoleum and resilient floor coverings of vinyl, linoleum, and similar materials                                                          |
| 41 | Taps, valves, and fittings for sinks, washbasins, bidets, toilets, baths, and similar sanitary equipment; central heating radiator valves |
| 42 | Hard coal and lignite                                                                                                                     |
| 43 | Wooden-framed seating furniture                                                                                                           |
| 44 | Wooden dining room and living room furniture                                                                                              |
| 45 | Specialized seating furniture with metal frames                                                                                           |
| 46 | Multiple-walled insulating glass units and glass mirrors                                                                                  |
| 47 | Carpets and rugs                                                                                                                          |
| 48 | Household refrigerators and freezers                                                                                                      |
| 49 | Household washing and drying machines                                                                                                     |
| 50 | Household vacuum cleaners                                                                                                                 |
| 51 | Oil-filled radiators, electric convectors, and fan heaters                                                                                |
| 52 | Heat-exchange equipment, refrigeration equipment, and air-conditioning equipment                                                          |
| 53 | Mixers, food processors, and juice extractors                                                                                             |
| 54 | Medical and dental instruments, therapeutic devices, prosthetic and orthopedic appliances                                                 |
| 55 | Passenger motor vehicles                                                                                                                  |
| 56 | New pneumatic rubber tires                                                                                                                |
| 57 | Motor fuels (including aviation gasoline)                                                                                                 |
| 58 | Gas oils (diesel fuel)                                                                                                                    |
| 59 | Television receivers, whether or not combined with radio receivers or sound/video recording and reproducing apparatus                     |
| 60 | Video cameras, video recording and playback equipment, and digital cameras                                                                |
| 61 | Computers and related equipment, parts, and accessories                                                                                   |
| 62 | Notebooks and exercise books                                                                                                              |
| 63 | Shampoos, hair lacquers, and hair styling preparations                                                                                    |
| 64 | Toothpaste and tooth-cleaning powders                                                                                                     |
| 65 | Soap and organic surface-active products for use as soap; paper, wadding, felt, and related products                                      |
| 66 | Shaving preparations, deodorants and antiperspirants, bath preparations, perfumes, cosmetics, and toilet preparations                     |
| 67 | Toilet paper                                                                                                                              |
| 68 | Sanitary towels, tampons, baby diapers, nappies, and similar sanitary products and apparel articles of paper pulp and paper               |
| 69 | Clocks and watches, excluding clock and watch movements and parts                                                                         |