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Abstract

This study focuses on estimating micro-level parameters to support the calibration of a dynamic stochastic general equilibrium (DSGE) model for the Kazakhstani economy.

We estimate several key structural parameters, including the output elasticities of labor and capital, the depreciation rate of fixed capital, and the Frisch elasticity of labor supply. The analysis is based on enterprise-level microdata from 2009 to 2024 and employs econometric techniques.

The resulting estimates improve the realism of macroeconomic models, align them more closely with national data, and provide a solid basis for assessing how the economy responds to various shocks and policy measures.

Keywords: capital elasticity, labor elasticity, depreciation rate, Frisch elasticity

JEL Classification: E22, E23, E24, C51

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1. ESTIMATION OF CAPITAL AND LABOR ELASTICITIES

1.1 Introduction

The estimation of capital and labor elasticities in aggregate output represents a fundamental component of macroeconomic analysis. These parameters describe the distribution of value added across production factors and provide insights into the underlying structure of the production process. Reliable estimates of factor elasticities are crucial for calibrating the production function in dynamic stochastic general equilibrium (DSGE) models, as they shape the responses of output, employment, and investment to macroeconomic shocks.

For Kazakhstan, obtaining such estimates is particularly relevant given the structural heterogeneity of the economy, the significant share of labor-intensive industries, and the variation in capital intensity across sectors. Empirically grounded production-function parameters allow for a more accurate representation of output dynamics and enhance the predictive performance of macroeconomic models.

In the empirical literature, capital and labor elasticities are commonly estimated using either aggregate national accounts data or firm-level microdata, which facilitates cross-country and cross-sector comparisons. This study employs the latter approach - using enterprise-level data for Kazakhstan - to derive more precise estimates that reflect actual differences in the economy's production structure.

1.2 Methodology

The Cobb–Douglas production function with constant returns to scale is a standard framework in economic theory for analyzing the contributions of labor and capital to output. A central empirical task is to obtain consistent estimates of factor elasticities.

Conventional ordinary least squares (OLS) estimates are subject to endogeneity bias, as firms choose input levels conditional on their individual productivity. To address this issue, the analysis employs the Wooldridge (2009) estimator based on the generalized method of moments (GMM).

This approach is a modification of the control-function method of Olley and Pakes (1996) and offers several advantages:

- it resolves the identification problem highlighted by Ackerberg et al. (2006);
- it allows for heteroskedasticity and serial correlation in the error term;
- it provides robust standard errors.

The model is specified as follows:

$$y_{it} = \alpha_0 + \alpha_l l_{it} + \alpha_k k_{it} + c_{it}\lambda + \varepsilon_{it},$$

y_{it} - denotes output,
l_{it} - labor (measured by the wage)
k_{it} - capital (fixed assets),
$c_{it}\lambda$ - an approximating function controlling for unobserved productivity, and
ε_{it} - represents a sequence of shocks that is assumed to be conditionally independent of current and past input choices.

Lagged values of capital, labor, and intermediate inputs are used as instruments, while the function c_{it} is specified as a second-order polynomial in capital and material inputs.

1.3 Data

Annual enterprise-level data for small, medium, and large firms (forms 2-MP, 1-PF) for the period 2009–2024, provided by the Bureau of National Statistics of Kazakhstan, were used for the analysis. Banks, insurance companies, educational and healthcare institutions, as well as public associations, were excluded from the sample.

The final dataset comprises 305,793 observations over the entire period.

Monetary values were converted to real terms using the corresponding deflators. Observations with missing, zero, or negative values of key variables were excluded.

Table 1.1. Distribution of observations by sector (2009–2024)

Sector	**Number of observations**	**Share, %**
Agriculture	27492	8.99
Mining	21924	7.17
Manufacturing	37657	12.31
Construction	31602	10.33
Services	187118	61.19
Total	305793	100

Source: Bureau of National Statistics of Kazakhstan, ASPIR

Table 1.2. Distribution of observations by year (2009–2024)

Year	**Number of observations**	**Share, %**
2009	17878	5.85
2010	22563	7.38
2011	15490	5.07
2012	15174	4.96
2013	20510	6.71
2014	21315	6.97
2015	20901	6.84
2016	22938	7.50
2017	23173	7.58
2018	21319	6.97
2019	19407	6.35
2020	17335	5.67
2021	17258	5.64
2022	17910	5.86
2023	15362	5.02
2024	17260	5.64
Total	30793	100

Source: Bureau of National Statistics of Kazakhstan, ASPIR

Table 1.3. Descriptive statistics of log-transformed variables (2009–2024)

Variable	Observations	Mean	Std. Dev.	Min	Max
Log of output	305793	10.58	2.36	2.20	18.34
Log of capital	305793	9.07	3.08	-1.16	18.30
Log of wage bill	305793	9.41	2.00	4.08	16.33
Log of material inputs	261041	9.32	2.83	1.35	17.66

Source: Bureau of National Statistics of Kazakhstan, ASPIR

1.4 Results

The Cobb–Douglas production function was estimated using the Wooldridge (2009) approach on a pooled sample of small, medium, and large firms in Kazakhstan for the period 2009–2024. The results are presented in Table 1.4.

Table 1.4. Labor and capital elasticities for firms in Kazakhstan, 2009–2024

Sector	Labor Elasticity	Capital Elasticity	Pseudo R^2	N (observations)
All sectors	**0.71*** **	**0.29*** **	0.718	152973
Agriculture	0.50***	0.50***	0.759	17382
Mining	0.69***	0.31***	0.805	9732
Manufacturing	0.55***	0.45***	0.846	21634
Construction	0.58***	0.42***	0.761	14487
Services	0.78***	0.22***	0.656	86791

*Note: *** indicates significance at the 1% level.*
Source: Authors' calculations

The estimated elasticities indicate that labor remains the dominant factor of production in the Kazakhstani economy, reflecting its structure, where a significant share is attributed to labor-intensive activities such as services. Overall, the labor elasticity of aggregate output is approximately 0.71, while the capital elasticity is around 0.29.

The highest labor elasticities are observed in the services sector (0.78) and the mining sector (0.69), while the lowest are in manufacturing (0.55) and agriculture (0.50). This distribution reflects differences in sectoral production structures:

- In the services sector, a high share of wages corresponds to the predominance of human capital and relatively low capital intensity;
- In manufacturing and construction, significant capital elasticity reflects the necessity of investment in equipment and fixed assets;
- Agriculture demonstrates a relatively balanced factor composition, which is typical for a sector with diverse technologies and forms of ownership.

The estimated labor and capital elasticities fall within the range commonly reported in empirical studies based on the Cobb–Douglas production function. According to international literature, capital elasticity in aggregate output typically ranges between 0.30 and 0.35 in both developed and developing economies (Adolfson et al., 2007; Dai et al., 2015; International Monetary Fund, 2016; Bhattarai & Trzeciakiewicz, 2017; Andreyev, 2020; Chen et al., 2023). Therefore, the obtained results are consistent with existing estimates and confirm the adequacy of the approach for calibrating DSGE model parameters.

To assess the robustness of the results, labor and capital elasticities were also estimated on several subsamples: separately for medium and large firms, as well as for small firms. For small firms, due to the availability of data on the number of employees, an alternative calculation of labor elasticity was performed using headcount instead of the wage bill. All results are presented in the Appendix (Tables 1A–3A) and generally confirm the stability of the estimates across the main sectors of the economy.

1.1 Conclusion

The estimation of the Cobb–Douglas production function using the Wooldridge (2009) methodology indicates that labor has remained the primary factor of production in the Kazakhstani economy over the period 2009–2024. The average labor elasticity of output is approximately 0.71%, while the capital elasticity is 0.29. This reflects the predominance of labor-intensive sectors, particularly services, in the country's economic structure.

Inter-sectoral differences highlight the presence of structural heterogeneity: labor elasticity reaches 0.78 in the services sector, whereas capital elasticity is higher, around 0.45–0.50, in manufacturing and agriculture. These results are consistent with the technological characteristics and capital intensity of the respective sectors.

Overall, the findings confirm that Kazakhstan's economic growth relies heavily on labor, while improvements in labor productivity and technological upgrading of capital remain key avenues for strengthening growth potential.

2 ESTIMATION OF THE DEPRECIATION RATE OF FIXED CAPITAL

2.1 Introduction

The depreciation rate measures the rate at which fixed capital loses value and exits the productive stock, indicating the speed at which physical assets depreciate. Accurate estimation of this parameter is critical for modeling capital accumulation and investment dynamics in macroeconomic frameworks, including DSGE models, where depreciation affects both the stability of equilibrium and the economy's response to shocks.

In the context of Kazakhstan, estimating the depreciation rate is particularly relevant due to the composition of fixed capital, which includes a substantial share of machinery and buildings acquired at different stages of investment cycles. This leads to heterogeneity in physical and economic depreciation rates, as well as varying sensitivity of investment activity to macroeconomic conditions. While the literature commonly relies on aggregate national accounts data to estimate depreciation rates, firm-level microdata allows for more precise and sector-specific estimates.

This section aims to estimate the average depreciation rate of fixed capital in the Kazakhstani economy using enterprise-level data. The resulting estimates will be applied in DSGE model calibration to improve the realism of capital and investment dynamics.

2.2 Methodology

To estimate the depreciation rate of fixed capital for small firms, the Perpetual Inventory Method (PIM) is employed, which is a standard tool for analyzing capital accumulation and depreciation dynamics. The evolution of the capital stock is described by the following equation:

$$K_t = (1 - \delta)K_{t-1} + I_t,$$

where K_t is the capital stock in period t, I_t denotes investment in fixed capital, and δ - is the depreciation rate.

Following the approach of Schündeln (2013), the equation is transformed into a logarithmic form suitable for econometric estimation:

$$\log(K_t - I_t) - \log(K_{t-1}) = \log(1 - \delta) + \varepsilon_t,$$

where ε_t is a stochastic error term.

The parameter $\log(1 - \delta)$ is estimated using pooled ordinary least squares (OLS) and fixed-effects (FE) models. To control for heterogeneity across years and industries, and to mitigate the impact of potential measurement errors, the model incorporates temporal, industry, and regional fixed effects:

$$\log(K_{it} - I_{it}) - \log(K_{it-1}) = c + year_i + industry_{it} + location_{it} + \varepsilon_{it}$$

where $year_i$, $industry_{it}$ and $location_{it}$ denote year, industry, and regional fixed effects, respectively, and the constant $c = log(1 - \delta)$.

Once the constant c is estimated, the depreciation rate is computed as:

$$\delta = 1 - \exp(c)$$

This approach allows for comparison of fixed capital depreciation dynamics across sectors and time periods, as well as for identifying structural characteristics of small firms in terms of the intensity of capital utilization and renewal.

2.3 Data

The analysis uses data on small firms in Kazakhstan obtained from the Bureau of National Statistics of the Republic of Kazakhstan. The primary source is form "2-MP" on the economic activity of small enterprises, covering the period 2009–2024.

The dataset includes the following information:

- Fixed capital (K_t) – the value of fixed assets at the end of the year;
- Investment (I_t) – capital expenditures on the acquisition and modernization of fixed assets during the year;
- Industry and regional affiliation.

The sample includes only small firms, defined according to national criteria (e.g., firms with fewer than 100 employees). Observations with missing or incorrect values were excluded, and all monetary variables were converted to constant prices to ensure comparability across years.

2.4 Variables and construction

Table 2.1. Variable

Variable	Notation	Definition	Unit of Measurement
Fixed capital	K_t	Value of fixed assets at the end of the year	thousand KZT, in constant prices
Investment	I_t	Annual expenditures on fixed assets	thousand KZT, in constant prices
Industry dummy variables	industry	Agriculture, industry, services, etc.	categorical
Time dummy variables	year	Yearly effects	categorical
Regional dummy variables	location	Контроль местоположения	categorical

All monetary variables are deflated using the gross fixed capital formation deflator. Observations where $K_t - I_t \leq 0$ are excluded to avoid logarithms of zero or negative values. Additionally, all quantitative variables are winsorized at the 1st and 99th percentiles to mitigate the impact of outliers.

2.5 Results

The estimated results in Table 2.2 indicate that the depreciation rate of fixed capital is 13.5%, calculated as $1-e^{-0,148}$=0.135) using the OLS method. After introducing fixed effects (e.g., for years, sectors, or firms), the estimated depreciation rate rises to 18.8% ($1-e^{-0,209}$=0.188). This suggests that, on average, firm capital depreciates by approximately 13.5–18.8% per year. The higher estimate after including fixed effects indicates that accounting for individual effects better captures the actual wear and tear of capital, which is underestimated without them.

Table 2.2. Regression results

Variable	$\log(K_{it} - I_{it}) - \log(K_{it-1})$	$\log(K_{it} - I_{it})$
Constant	**-0.148*** (0.0097)**	**-0.209*** (0.028)**
K_{it-1}	-	1.008 (0.001)
Year fixed effects	No	Yes
Industry fixed effects	No	Yes
Regional fixed effects	No	Yes
Observation	298946	227750
R^2	0,000	0,9217

Source: Authors' calculations

*Note: Standard errors are reported in parentheses. *** indicates significance at the 1% level.*

In the second regression, K_{t-1} represents the value of fixed capital at the beginning of the period, while in the first regression it corresponds to the lagged capital stock.

It is also worth noting that in Adilkhanova (2020), using microdata for enterprises over 2015–2018 (form No.11, around 10,000 medium and large firms, totaling 32,234 observations), a somewhat different estimate of the depreciation rate was obtained. According to those results, the depreciation rate was 8.3% using a random-effects model. After including industry fixed effects, the depreciation rate ranged from 7.2% in the services sector to 10.9% in agriculture and 15.3% in construction, reflecting structural differences across sectors.

The present study uses a sample of small firms covering a significantly longer period and a broader set of enterprises. Differences in estimates are partly explained by this expansion of the dataset. As new microdata become available, a reassessment of the depreciation rate is planned to ensure comparability of results and their subsequent use in DSGE models for Kazakhstan.

2.6 Alternative approach

To further assess the robustness of the results, the depreciation rate for each firm i in year t was calculated directly as the ratio of depreciation to the value of fixed capital at the end of the period:

$$\delta_{it} = \frac{depreciation_{it}}{fixed\ capital_{it}}$$

Prior to calculation, all monetary values were converted to real terms using base-year prices and winsorized at the 1st and 99th percentiles to eliminate extreme outliers and accounting errors. Observations with δ_{it}>1 were excluded as economically implausible, since depreciation cannot exceed 100% of an asset's value within a year.

Subsequently, the depreciation rate is controlled for firm heterogeneity using the following model:

$$\delta_{it} = c + \alpha_i + year_{it} + industry_{it} + location_{it} + \epsilon_{it}$$

where δ_{it}- is the firm-level depreciation rate,
c is the constant,
α_i denotes firm fixed effects,

$year_{it}, industry_{it}$ and $location_{it}$ - represent year, industry, and regional fixed effects, respectively,
ϵ_{it} is the error term.

Table 2.3. Regression results with fixed effects

Variable	**OLS**	**Fixed effects**
Constant	**0.153*** (0.007)**	**0.227*** (0.026)**
Year fixed effects	Да	Да
Industry fixed effects	Да	Да
Regional fixed effects	Да	Да
Observation	Нет	Да
Year fixed effects	206393	206393

Source: Authors' calculations
*Note: Standard errors are reported in parentheses. *** indicates significance at the 1% level.*

The regression results indicate that the average depreciation rate of fixed capital is approximately 15.3% using OLS and 22.7% when firm fixed effects are included. The increase in the estimated depreciation rate after accounting for individual firm effects reflects significant heterogeneity in capital wear and tear across enterprises.

2.7 Conclusion

The estimation of the depreciation rate of fixed capital using microdata from small enterprises in Kazakhstan provides a quantitative assessment of capital wear and reveals important structural characteristics. Application of the Perpetual Inventory Method (PIM) shows that the average annual depreciation rate ranges between 13.5% and 18.8%. This level reflects more intensive utilization of fixed assets in the small-enterprise sector compared with earlier estimates for medium and large firms.

To verify the robustness of the results, an alternative approach based on firm-level depreciation ratios was employed. The findings from this model confirm the main conclusions: the average depreciation rate is approximately 15.3% under OLS and increases to 22.7% when firm fixed effects are included. This indicates substantial heterogeneity across enterprises, driven by differences in capital intensity, the age structure of equipment, and sectoral characteristics.

A comparison with previous estimates (Adilkhanova, 2020) shows that depreciation rates for medium and large enterprises were significantly lower, ranging from 7% to 15% depending on the sector. The higher values obtained in the present study may be explained both by differences in the sample composition and by the extended time coverage of the dataset. It should also be noted that most DSGE models commonly use a standard depreciation rate of about 10% per year (Christiano et al., 2005; Altig et al., 2005; Smets and Wouters, 2007), which aligns with assumptions widely adopted in the literature.

Future work will involve updating the estimates as the microdata base expands.

3 ESTIMATION OF THE FRISCH ELASTICITY

3.1 Introduction

The Frisch elasticity of labor supply is a structural parameter widely used in macroeconomic models to characterize the responsiveness of households' labor supply to changes in real wages, holding the marginal utility of wealth constant. It reflects the extent to which individuals adjust their hours worked in response to changes in labor compensation, under otherwise comparable conditions.

In the context of DSGE models, the Frisch elasticity enters the household utility function and governs labor–leisure trade-offs, thereby shaping household behavior in the labor market. The parameter captures the pure substitution effect: when wages rise, labor becomes relatively more attractive than leisure, prompting households to increase their labor supply.

Estimating this parameter makes it possible to quantify the degree of flexibility with which the population responds to shocks that affect real wages. Accounting for the specific features of Kazakhstan's labor market enhances the realism and empirical relevance of model outcomes that rely on this parameter.

3.2 Methodology

The Frisch elasticity reflects the responsiveness of labor supply to predictable changes in wages while holding the marginal utility of wealth constant. Accurate estimation of this parameter requires isolating the pure substitution effect – namely, workers' response when an additional hour of labor becomes relatively more rewarding than an hour of leisure.

Observed wage fluctuations contain both predictable and unpredictable components. Unanticipated changes generate not only substitution effects but also income effects: a sudden increase in wages makes individuals wealthier, which may reduce their willingness to supply labor. Predictable changes, however, are incorporated into expectations and therefore do not affect perceived wealth; instead, they capture only the incentive-driven component of labor supply. The key objective of estimating the Frisch elasticity is thus to identify this substitution-based response.

Standard OLS estimation may suffer from endogeneity bias. This arises because unexpected wage shocks correlate with the labor supply equation's error term: workers may adjust their hours in response to transitory changes in income. As a result, one of the core OLS assumptions is violated.

Additionally, the literature (Altonji, 1986) highlights the issue of denominator bias when hourly wages are constructed as earnings divided by hours worked. Measurement errors in hours mechanically generate a spurious negative correlation between wages and hours, leading to downward-biased and theoretically inconsistent OLS estimates. In the baseline specification, aggregated measures (total wage fund and total hours worked) are used, reducing the likelihood of such distortions.

To address endogeneity, instrumental variables are employed. An appropriate instrument must affect wage changes while remaining uncorrelated with unanticipated shocks influencing labor supply. Following the classical approach of MaCurdy (1981) and Altonji (1986), the lagged change in wages is used as an instrument, capturing the predictable component of wage dynamics. Under this approach, current wage changes are related to past wage movements but remain orthogonal to short-term shocks affecting labor supply in the current period.

The empirical model is estimated using panel data with fixed effects, allowing for control of time-invariant firm and worker characteristics (such as productivity, sectoral affiliation, and organizational structure). The baseline equation, consistent with MaCurdy (1981) and Altonji (1986), is specified as:

$$\Delta ln(h_{it}) = \alpha + \beta\Delta ln(w_{it}) + \gamma\Delta ln(y_{it}) + \lambda_{it} + u_{it},$$

where $\Delta ln(h_{it})$ - percentage change in hours worked (labor supply),
$\Delta ln(w_{it})$ - percentage change in wages,
$\Delta ln(y_{it})$ - change in firm output (controls for labor demand),
λ_{it} - vector of time, industry, and regional dummies,
u_{it} - error term.
To eliminate endogeneity in $\Delta ln(w_{it})$, the variable is instrumented using its lagged value $\Delta ln(w_{i,t-1})$.

The model is estimated using fixed effects with firm-clustered standard errors. The coefficient on the instrumented wage variable (β) is interpreted as the Frisch elasticity, capturing the behavioral response of labor supply to predictable wage changes.

3.3 Data

The empirical analysis is based on annual data from Form 2-MP, "Report on the activities of small enterprises," covering the period from 2009 to 2024. The sample includes all small enterprises (with fewer than 100 employees), excluding banks, insurance companies, educational and medical institutions, as well as public associations.

The estimation relies on differenced variables, with the lag of the differenced wage variable used as an instrument. Since constructing differences and lags requires at least three consecutive annual observations, the effective estimation sample begins in 2011.

Prior to the analysis, all quantitative variables were winsorized at the 1st and 99th percentiles. This procedure mitigates the influence of extreme observations that may arise from reporting errors or isolated anomalies (such as abrupt jumps in total wage payments). Winsorization allows the full sample to be retained while limiting the impact of outliers on coefficient estimates and improving the robustness of the results.

All monetary variables are converted into real terms.

The final dataset contains 164,606 observations. It should be noted that differencing substantially reduces the effective sample size relative to the original panel because it retains only those enterprises that remain in the dataset for at least three consecutive years. This consideration is particularly relevant for small firms, some of which may exit the market earlier or transition into the category of medium and large enterprises.

For robustness, an alternative specification estimated in levels – without differencing the variables – is also reported.

Table 3.1. Variables

Variable	Notation	Mean	Standard Deviation	Minimum	Maximum
Total hours worked	h_{it}	35163.93	42851.57	32	187404
Wage fund (thousand KZT)	w_{it}	15285.33	26858.3	44.58	149017.9
Output (thousand KZT)	y_{it}	75914.87	180174.2	0	1136310
Time fixed effects	$year_{it}$	*categorical*	*categorical*	2009	2024
Regional fixed effects (region code)	$region_{it}$	*categorical*	*categorical*	11	75
Sector fixed effects	$sector_{it}$	*categorical*	*categorical*	1	20

In general, including regional and sectoral dummy variables is not strictly necessary because firm-level fixed effects already capture all time-invariant characteristics related to a firm's industry affiliation or regional location. Fixed effects remove the influence of factors that do not vary over time within each firm, such as sector-specific features or institutional differences across regions. However, firms may change their registered sector or region over the sample period. For this reason, regional and sectoral dummies are included in the empirical specifications. Time fixed effects are also included, as they control for aggregate macroeconomic shocks that affect all firms simultaneously.

3.4 Results

Table 3.2 presents the results of the Frisch elasticity estimation.

The first column of the table presents the estimation results based on the baseline specification, where all monetary variables are expressed in differences. These results capture within-firm variations and show the response of annual labor hours worked within each firm to changes in wages and output over time, holding other factors constant.

A 10% increase in wages is associated, on average, with a 5.7% increase in labor hours. The coefficient on output changes is also positive and statistically significant, controlling for variations in firm size. Time dummy variables are statistically significant as well, indicating the presence of common macroeconomic shocks affecting all firms in particular years.

An additional estimation including cross-effects (interactions between time and sector dummies to control for sector-specific shocks) was conducted for robustness check. The estimated coefficients remained unchanged, further confirming the stability of the results.

The second column reports the results estimated in levels as a robustness check. Comparison of the two specifications shows that the sign and statistical significance of the coefficients are preserved, although their magnitudes differ slightly. The differenced model captures the short-term response of labor supply to changes in wages and output, reflecting the adaptation of firms and workers to current economic conditions within the same period. The levels model with fixed effects describes the within-firm relationship between wages and employment that may evolve over time.

Table 3.2. Results

	(1) $\boldsymbol{D.ln(h_{it})}$	(2) $\boldsymbol{ln(h_{it})}$
(1) $\boldsymbol{D.ln(w_{it})}$**; (2)** $\boldsymbol{ln(w_{it})}$	0.572*** (0.012)	0.673*** (0.005)
(1) $\boldsymbol{D.ln(y_{it})}$; **(2)** $\boldsymbol{ln(y_{it})}$	0.035*** (0.0045)	0.032*** (0.003)
Time effects	Yes***	Yes***
Sector effects	Yes	Yes
Regional effects	Yes	Yes
Constant	0.083 (0.138)	40.13*** (0.092)
$\boldsymbol{R^2}$ (between)	0.23	0.71
$\boldsymbol{R^2}$ (overall)	0.11	0.64

Source: Authors' calculations.
Note: Standard errors are reported in parentheses.
**** indicates statistical significance at the 1% level.*
The second column of the table presents the results estimated in levels.

Thus, the estimates in levels can be interpreted as an approximation of the long-run elasticity. The consistency of coefficient signs and the similarity in magnitude between specifications confirm the robustness of the results and their alignment with theoretical expectations.

3.5 Alternative estimation

3.5.1 Estimation using per-worker averages

In this section, we estimate the model on a per-worker basis. This approach allows focusing on labor intensity, i.e., how the average hours worked per employed individual respond to changes in wages and output. In this specification, all variables (hours worked, wage fund, and output) are normalized by the actual number of employees rather than the headcount. Using the actual number of employees is more appropriate, as the headcount includes workers temporarily absent from the workplace (e.g., on vacation, maternity leave, or unpaid leave) and therefore does not accurately reflect the volume of labor actually engaged.

The coefficients in this model capture the response of the average worker to changes in wages and output, whereas the baseline specification characterizes the aggregate labor supply response in the economy.

Table 3.3. Results

	(1)$\boldsymbol{D.ln(h_{it}/workers_{it})}$	**(2)**$\boldsymbol{ln(h_{it}/workers_{it})}$
(2) $\boldsymbol{D.ln(w_{it})}$; **(2)** $\boldsymbol{ln(w_{it})}$	0.282*** (0.013)	0.280*** (0.0068)
(2) $\boldsymbol{D.ln(y_{it})}$; **(2)** $\boldsymbol{ln(y_{it})}$	0.029*** (0.003)	0.021*** (0.002)

Time effects	Yes	Yes***
Sector effects	Yes	Yes
Regional effects	Yes	Yes
Constant	0.151 (0.105)	50.82*** (0.076)
$\boldsymbol{R^2}$ (between)	0.01	0.054
$\boldsymbol{R^2}$ (overall)	0.01	0.030

Source: Authors' calculations.
Standard errors in parentheses.
**** denotes statistical significance at 1%.*
The second column of the table presents the results estimated in levels.

Table 3.3 presents the results of the per-worker estimation, which allows analyzing how average hours worked per employee change in response to fluctuations in the average wage.

The first column of the table reports the results from the model in differences. The coefficient on the change in wages per worker indicates that a 10% increase in wages per worker is associated with an approximate 2.8% increase in average hours worked per employee. This positive and statistically significant coefficient confirms the presence of a positive response of labor intensity to predictable wage increases, reflecting the substitution effect while controlling for firm size.

The second column presents the estimates in levels. The signs and magnitudes of the coefficients remain consistent, indicating the robustness of the results.

The coefficients calculated on a per-worker basis are lower than those in the baseline specification, which aligns with the interpretation that part of the aggregate labor supply response occurs through changes in the number of employees rather than through hours worked by existing employees. This confirms that individual workers respond to wage changes, but the effect is more moderate than the overall labor supply response.

3.5.2 Estimation of Frisch elasticity using employment levels

The specification in which the dependent variable is the actual number of employees allows us to assess how changes in expected wages affect individuals' decisions to enter or exit employment. In other words, it captures the extent to which higher expected wages incentivize the participation of additional labor

Table 3.4 presents the results of the Frisch elasticity estimation, with the dependent variable defined as the logarithm of the actual number of employees.

The first column reports the results from the model in differences. The estimates indicate that a 10% increase in wages is associated with an approximate 4.7% increase in employment.

The second column presents the results of the alternative estimation using variables in levels. The coefficient on wages is somewhat higher than in the short-term specification, indicating that over a longer horizon, wage increases are associated with a stronger effect on employment.

Overall, the estimates are consistent with theoretical expectations: the positive and statistically significant relationship between wages and employment confirms the presence of a substitution effect.

Table 3.4. Results

	(1) $D.ln(h_{it}/workers_{it})$	(2) $ln(h_{it}/workers_{it})$
(1) $D.ln(w_{it})$; (2) $ln(w_{it})$	0.47*** (0.007)	0.59*** (0.004)
(1) $D.ln(y_{it})$; (2) $ln(y_{it})$	0.010*** (0.001)	0.015*** (0.001)
Time effects	Да	Да***
Sector effects	Да	Да
Regional effects	Да	Да
Constant	-0.009 (0.0068)	-20.61*** (0.064)
R^2 (between)	0.08	0.83
R^2 (overall)	0.43	0.83

Source: authors' calculations.
Note: Standard errors are reported in parentheses.
**** indicates statistical significance at the 1% level.*
The second column of the table presents the results estimated in levels.

3.6 Conclusion

The estimation results confirm a positive response of labor supply to predictable changes in wages, consistent with theoretical expectations. The findings across different specifications align with the range of estimates typically observed in microdata studies (MaCurdy, 1981; Altonji, 1986). Similar values of the Frisch labor supply elasticity (around 0.5) are employed in contemporary DSGE models for various countries (Smets and Wouters, 2007; Chen et al., 2023; Fernández-Villaverde et al., 2015).

Comparison with Adilkhanova (2020) shows that estimates for small enterprises are lower than those for medium and large firms. This may reflect lower labor supply flexibility in small firms, where employment is less responsive to wage changes. Additionally, differences in the time coverage may capture structural shifts in the labor market.

Updates to the estimates will be carried out as additional data become available.

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5 Appendix

Table 1A. Labor and capital elasticities for medium and large firms in Kazakhstan, 2009–2024

Sector	Labor elasticity (wage fund)	Capital elasticity	N
All sectors	0.75***	0.25***	56,743
Agriculture	0.64***	0.36***	6,079
Mining	0.67***	0.33***	2,746
Manufacturing	0.57***	0.43***	10,841
Construction	0.62***	0.38***	7,299
Services	0.86***	0.14***	29,353

Source: Authors' calculations.

Table 2A. Labor and capital elasticities for small firms in Kazakhstan, 2009–2024

Sector	Labor elasticity (wage fund)	Capital elasticity	N
All sectors	0.73***	0.27***	90,280
Agriculture	0.50***	0.50***	10,764
Mining	0.73***	0.27***	6,755
Manufacturing	0.62***	0.38***	9,910
Construction	0.64***	0.36***	6,379
Services	0.79***	0.21***	54,285

Source: Authors' calculations.

Table 3A. Labor and capital elasticities for small firms in Kazakhstan (using number of employees), 2009–2024

Sector	Labor elasticity (employment)	Capital elasticity	**N**
All sectors	0.71***	0.29***	79,601
Agriculture	0.47***	0.53***	9,260
Mining	0.72***	0.28***	6,301
Manufacturing	0.68***	0.32***	8,643
Construction	0.71***	0.29***	5,529
Services	0.73***	0.27***	47,774

Source: Authors' calculations.